A Technical Analysis of the Extra Coil as a Slow Wave Helical Resonator

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Abstract

In this paper, we present an analytical development of the Extra Coil as a top-loaded slow-wave helical resonator. Our treatment starts with a foreshortened coaxial resonator whose inner conductor is constructed of an end loaded spiral delay line. We develop formulas for the slow wave velocity factor and characteristic impedance of the equivalent transmission line which are valid as the outer walls of the resonator recede to infinity – leaving Tesla's 'open coil' resonator above ground.

The Voltage Standing Wave Ratio (VSWR) is then analyzed, including losses, and the voltage step-up of any distributed Tesla Coil resonator is obtained. The resulting model not only predicts the resonant behavior of the Extra Coil, but readily permits its representation on a Smith Chart. Several examples of Extra Coil analyses are presented numerically, and displayed on Smith Charts. Specifically, the November 1, 1899 and January 2, 1900 Extra Coils of Tesla's Diary are discussed in detail and are shown to produce voltage step-ups on the order of 10 to 15 megavolts.

The model also serves to explain how Tesla tuned the RF portion of the Colorado Springs apparatus and is shown to be consistent with specific Diary instructions by Tesla.

Introduction

Without presenting any historical background, let us go directly to the Colorado Springs apparatus and in particular to the 'Extra Coil', which so spectacularly dominates the center stage of Tesla's amphitheater.

The authors believe that the most fruitful way to characterize the 'Extra Coil' is as a **distributed** circuit or loaded transmission line, and not in terms of lumped circuit elements. This is particularly enlightening when the structure is excited by time harmonic fields, i.e., in the sinusoidal steady state. The analysis can be extended to include damped wave excitation, as was used in Tesla's day. We hypothesize that Tesla operated the Extra Coil against the earth as a slow-wave resonant transmission line. (Note that it would be undesirable to use it as an antenna or as a loading coil for the tower since the radiation loss will decrease the input VSWR and seriously reduce its ability to step-up the base-fed excitation.)

Consider the foreshortened coaxial resonator, so familiar in microwave systems, shown in Figure l-a.
In order to make the resonator physically smaller, the center conductor is replaced by a spiral delay line (Figure 1-c). A ball is placed on the top for capacitive tuning and to prevent disruptive discharge. The walls then recede to infinity, leaving Tesla's open coil resonator (Figure l-d). This conceptual view of the Extra Coil, as shown in Figure 2, could now be analyzed in great detail if we only knew the velocity factor and effective characteristic impedance of the transmission line equivalent of Figure 1-e.

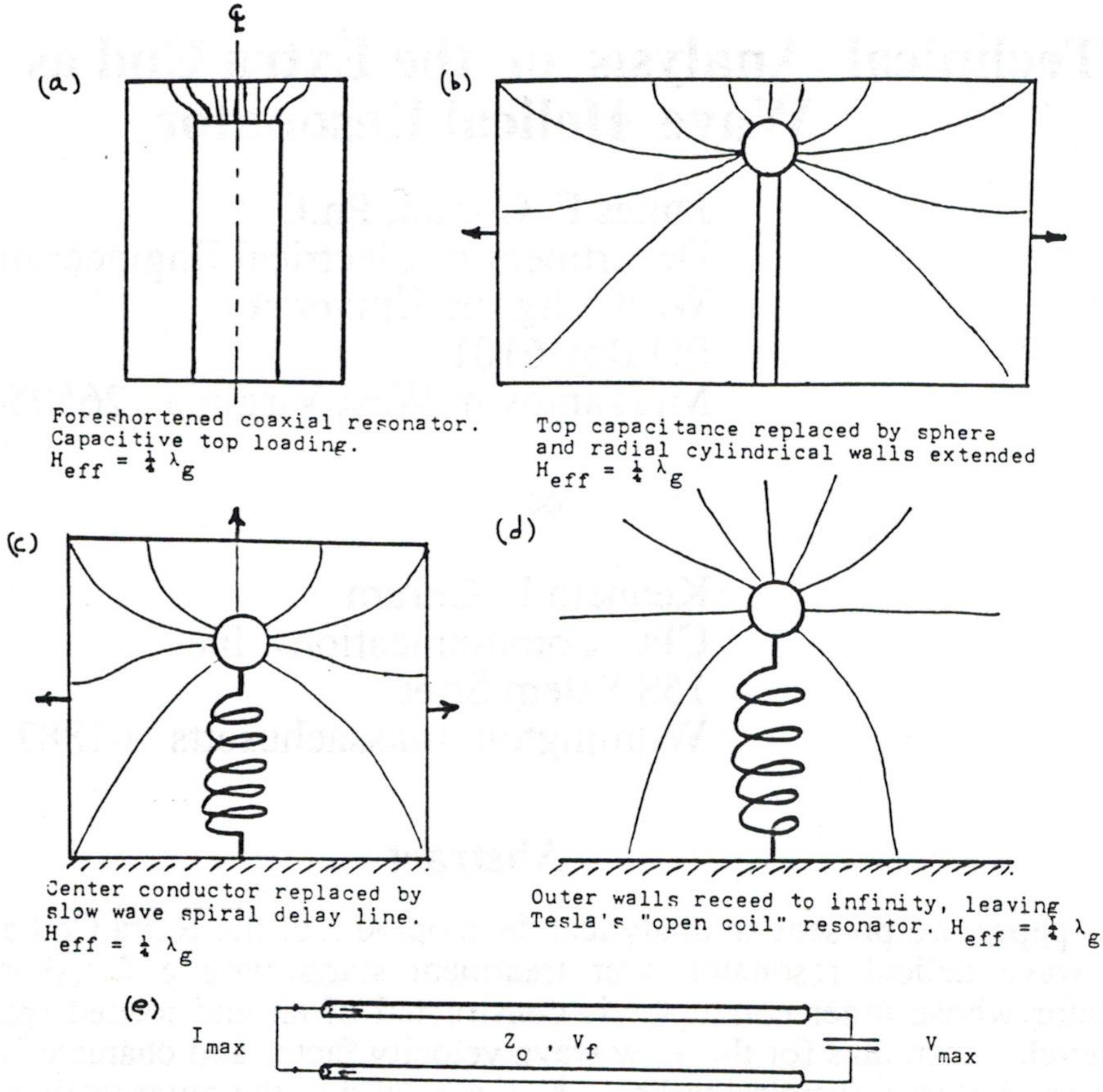

Figure 1. Conceptual development of Tesla's Extra Coil.
(a,b) The capacitive loaded foreshortened coaxial resonator.
(c) The slow wave top Ioaded resonator.
(d) The final open coil resonator used at Colorado Springs.
(e) Equivalent slow wave transmission line resonator.

Helix Characterization

Many years ago, Kandoian and Sichak analyzed the helix and obtained engineering formulas of great practical value.[1,2] Let us summarize their results:

1 "Wide Frequency Range Tuned Helical Antennas and Circuits", by A.G. Kandoian and W. Sichak, IRE Convention Record, 1953, Part 2 - Antennas and Communications, pp. 42-47. Reprinted in Electrical Communications, Vol. 30, December, 1953, pp. 294-299.

2 Reference Data for Radio Engineers, H.P. Westman (editor), 4th Edition, 1956, pp. 682-686.

1. The velocity factor for the speed of a wave disturbance **along the axis of the helix** is given by

$$V_f = \frac{v}{c} = \sqrt{1 + 20\left(\frac{D}{s}\right)^{2.5}\left(\frac{D}{\lambda_o}\right)^{1/2}} \quad (1)$$

where λ_0 = free space wavelength
D = helix diameter
s = turn-to-turn spacing
(all in the same units)

for

$$\frac{D^2}{s\lambda_o} \leq \frac{1}{5}$$

2. The ratio of the peripheral phase velocity of a wave along the helical wire to the speed of light is given by the expression

$$\frac{V_w}{c} \approx 1.25\left(\frac{H}{D}\right) 1/5 \quad (2)$$

where H = the physical height of the helix for

$$D \geq s \quad \text{and} \quad \frac{D^2}{s\lambda_o} \leq \frac{1}{5}$$

3. The structure's loss resistance, referred to the base, is given by the expression
4.

$$R_{Loss} = 12.5\frac{\left(\frac{V_w}{c}\right)}{d_w\sqrt{f_{MHz}}} \quad (3)$$

where d_w = wire diameter in inches.

5. Schelkunoff's transmission line effective characteristic impedance is

$$Z_o = \frac{60}{V_f}\left[\,l\,n\left(\frac{4H}{D}\right) - 1\right] \quad (4)$$

With these formulas, one can predict the behavior of a helical structure, with or without top loading, with surprising accuracy. In fact, one can even examine the voltage distribution along the top loaded helix. Not surprisingly, for a low-loss structure, the voltage at the top can rise to a considerable value, even for a relatively small excitation.

Tesla's Extra Coil Treated as a Top-Loaded Helical Transmission Line

When one looks at the pictures of the Colorado Springs extra coil, at first sight, it appears as though Tesla might have been using the extra coil as a base loading reactance for the tower. However, both from the 1899 Diary and from the Long Island notes, it is clear that the vertical mast was isolated from the extra coil. In the patents, it is referred to as 'an elevated insulated body of capacitance'. The only structure attached to the extra coil was the spherical ball at its top or various other rings or rods attached to the last turn of the coil.

We hypothesize that the extra coil structure was worked against ground as a resonant transmission line for the purpose of producing a voltage rise. The end loading capacitance is used to bring the structure into quarter-wave resonance with a voltage minimum at the base and voltage maximum $\lambda_g/4$ away – at the top of the Extra Coil. With either an open circuit or a capacitance at the top, the VSWR would only be limited by resistive losses and the V at the top could rise to a very high value.

Let us review the voltage step-up behavior of a slow-wave transmission line in the sinusoidal steady state. The analysis can be generalized to damped wave excitation in the usual manner if desired.

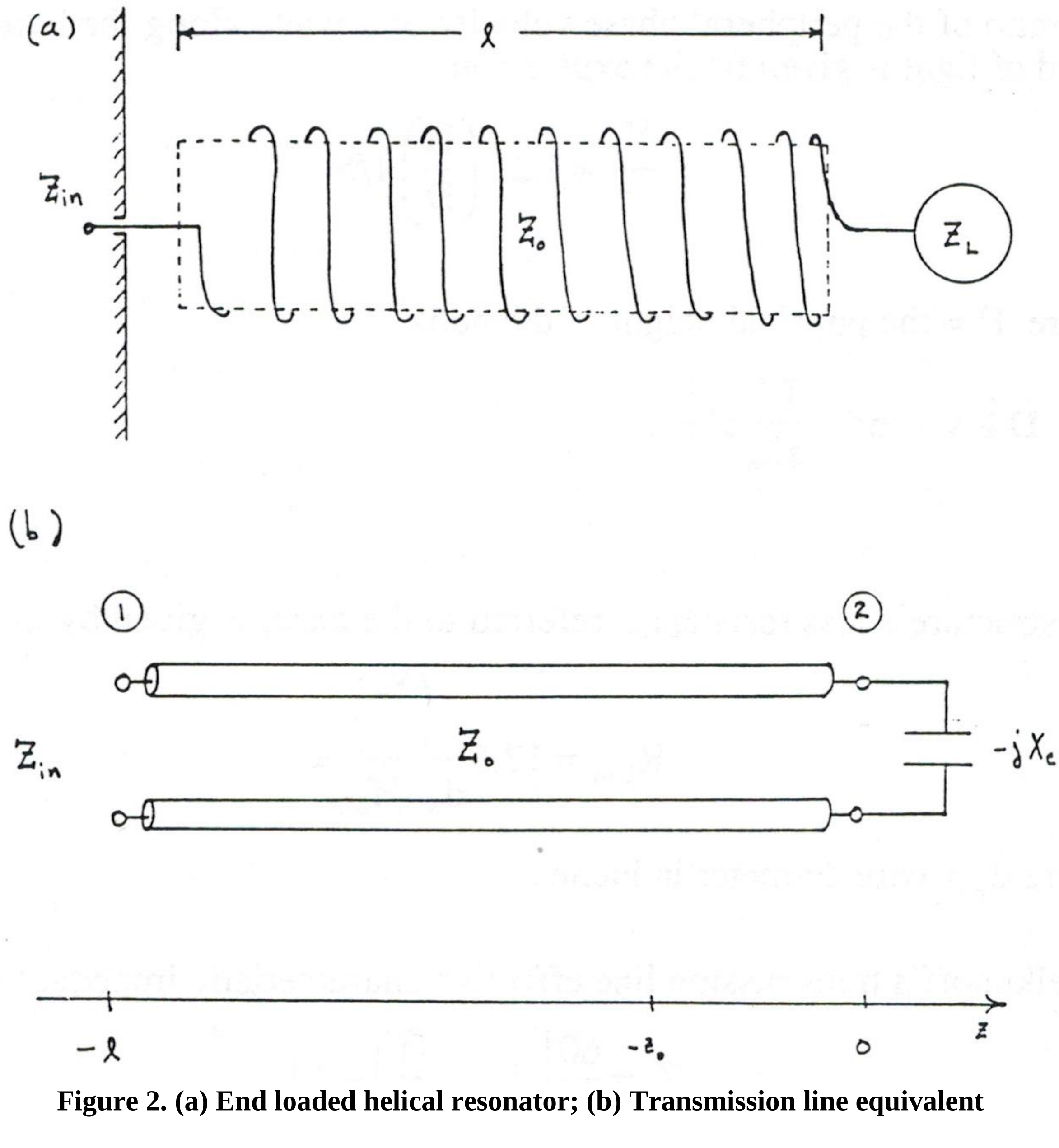

Figure 2. (a) End loaded helical resonator; (b) Transmission line equivalent

Voltage Step-Up on a Vertical Helical Resonator above a Conducting Ground Plane

Consider again, the top loaded helical delay line and its transmission line equivalent in Figure 2. The load impedance is simply

$$Z_L = \quad -j\,X_c = -j\,\frac{1}{(2\pi f C_T)} \qquad (5)$$

where C_T is effectively taken as the sum of the top ring capacitance (on the Extra Coil) plus the sphere to ground capacitance.

The solution of the transmission line equations leads to the voltage distribution along the line (or Extra Coil) as

$$V(z) = V_+ e^{\gamma z} + V_- e^{-\gamma z} \tag{6}$$

where γ is the complex propagation constant

$$\gamma = \alpha + j\beta \tag{7}$$

with phase constant

$$\beta = 2\pi/\lambda_g \tag{8}$$

where

$$\lambda_g = V_f \lambda_0 \tag{9}$$

The attenuation constant, α, is given below. The complex load reflection coefficient is given by

$$\Gamma_2 \stackrel{\Delta}{=} \frac{Z_L - Z_o}{Z_L + Z_o} = |\Gamma_2| \ \lfloor \Phi_2 \tag{10}$$

The electrical length of the line in degrees is given by

$$\theta = \beta H = \frac{360\,H}{V_f \lambda_o} \tag{11}$$

The complex reflection coefficient referred to the input end (or base) of the transmission line is given by

$$\Gamma_1 \stackrel{\Delta}{=} \frac{V_- e^{-\gamma l}}{V_+ e^{\gamma l}} = |\Gamma_2|\, e^{-2\alpha l} \ \lfloor \Phi - 2\,\theta \tag{12}$$

The VSWR follows from the definition

$$S \stackrel{\Delta}{=} \frac{V_{max}}{V_{min}} = \frac{|V_+| + |V_-|}{|V_+| - |V_-|} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \tag{13}$$

(V_{max} and V_{min} are spatially separated by one quarter wavelength.) On a lossless transmission line $|\Gamma_1| = |\Gamma_2|$ and S is equal to the maximum value which the impedance takes on along the line, normalized with respect to the characteristic impedance. On a lossy line the trajectories of $\Gamma e^{-\gamma l}$ are not circles but spiral inward on a Smith chart from Γ_2, as one approaches the input end of the transmission line from the load end. Consequently the VSWRs at the load end (2) and input end (1) differ as

$$S_1 = \tanh\left[\alpha\, l + \tanh^{-1}\left(\frac{1}{S_2}\right)\right] \tag{14}$$

For high standing wave ratios (i.e. S > 6:1) this expression may be approximated as

$$1/S_1 = 1/S_2 + \alpha l \tag{15}$$

with less than one percent error. Since the Extra Coil has a capacitive load, $S_2 \rightarrow$infinity and thus

$$S_1 \approx \frac{1}{\alpha l} \tag{16}$$

If the line were a full $\lambda_g/4$ long, then V_{max} would be equal to SV_{min} where V_{max} would occur at the top and V_{min} at the base. From equations (6) and (11) we may write the voltage at the base of the Extra Coil as

$$V_{base} = V_+ [e^{\alpha l} e^{j\theta} + |\Gamma_2| e^{-\alpha l} e^{j(\Phi - \theta)}] \tag{17}$$

Further, the voltage at the top of the Extra Coil may be expressed as

$$V_{Top} = V_+[1 + \Gamma_2] \tag{18}$$

Consequently

$$V_{Top} = \frac{V_{base} [1 + \Gamma_2]}{[e^{\gamma l} + \Gamma_2 e^{-\gamma l}]} \tag{19}$$

More specifically, for computational purposes,

$$|V_{Top}| = \frac{|V_{base}| \sqrt{[1 + |\Gamma_2| \cos\Phi]^2 + [|\Gamma_2| \sin\phi]}}{\sqrt{[e^{\alpha l} \cos(\Phi - \theta)]^2 + [e^{\alpha l} \sin\theta + e^{-\alpha l} |\Gamma_2| \sin(\Phi - \theta)]^2}} \tag{20}$$

where Φ is defined in Equation (10) and in Equation (11). lt should be obvious that the Extra Coil should be designed such that the denominator of Equation (20) is minimized. The only parameter left, to calculate the voltage step up, is the attenuation constant α.

Transmission lines are commonly presented through the vehicle of distributed circuit theory. The series inductance per unit length is taken as L henries per meter; the series loss resistance as R ohms per meter; the shunt distributed capacitance as C farads per meter; and the shunt leakage conductance as G siemans per meter.

The propagation attenuation constant then follows as

$$\alpha = \left(\frac{R}{2 Z_o} + \frac{G Z_o}{2} \right) \quad \text{Nepers/meter} \tag{21}$$

For Tesla's Extra Coil, we assume that (in the absence of discharges) the shunt coil-to-round conductance G is negligible. Consequently

$$\alpha = \frac{R}{2Z_o} \tag{22}$$

where R is the distributed copper loss. The parameter needed for Equation (20) is αl.

$$\alpha l = \frac{Rl}{2Z_o} = \frac{R_{Loss}}{2Z_o} \tag{23}$$

where R_{loss}, is the total loss resistance given by Equation (3) above. Thus

$$\alpha l = \frac{7.8125\,(\frac{H}{D})^{1/5}}{d_w Z_o \sqrt{f_{MHz}}} \tag{24}$$

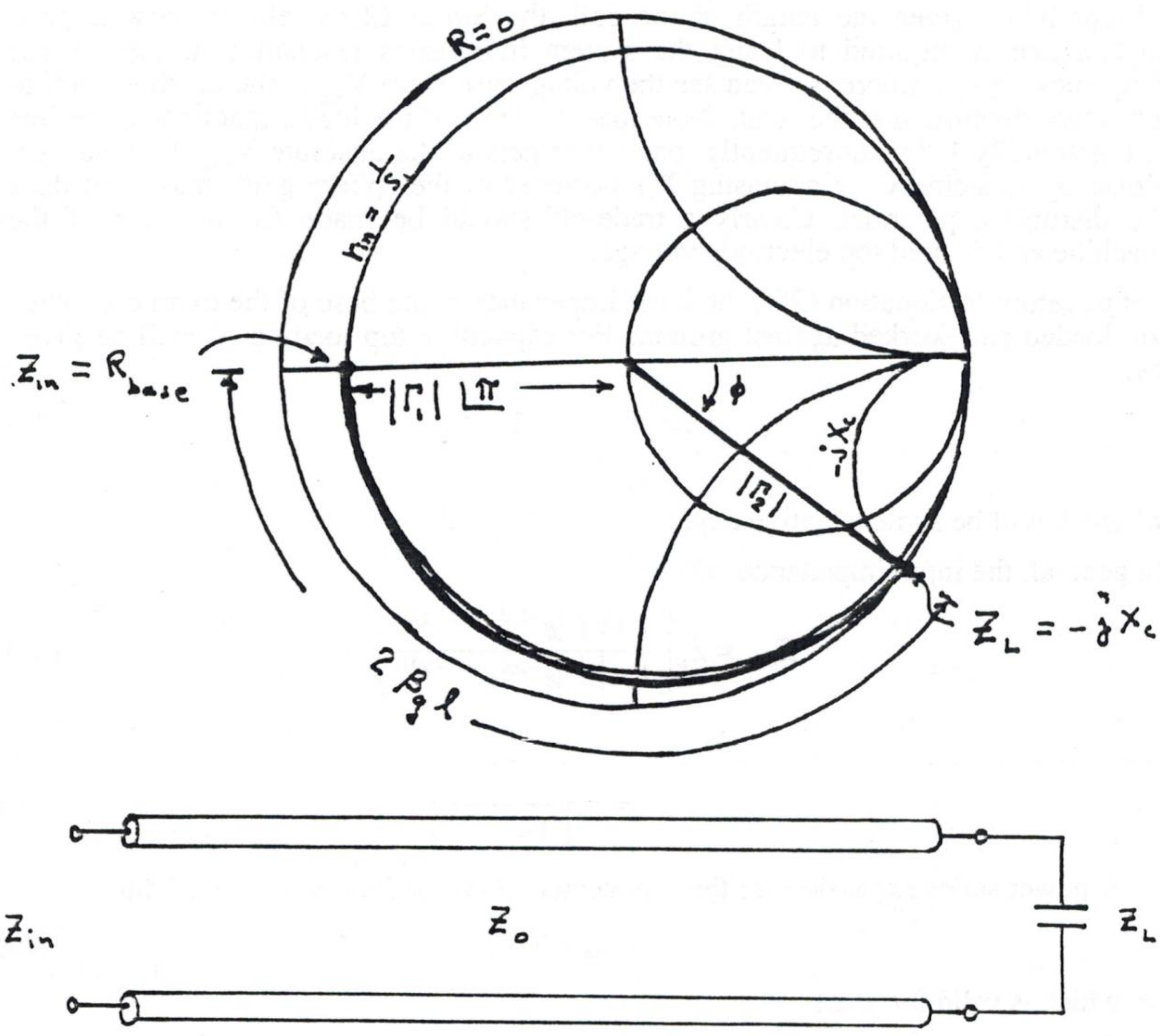

Figure 3. Representation of a top Ioaded Extra Coil on a Smith Chart.

Lastly, the input impedance, or impedance seen at the base when the top loaded coil is worked against ground, is given by

$$Z_{in} = Z_o \frac{Z_L + Z_o \tanh \gamma l}{Z_o + Z_L \tanh \gamma l} = Z_o \left[\frac{1 + \Gamma_2 \, e^{-2\gamma l}}{1 - \Gamma_2 \, e^{-2\gamma l}} \right] \tag{25}$$

where, for the low-loss case, Z is given by Equation (11).

The top loaded extra coil is clearly described by the Smith Chart shown in Figure 3. The load impedance is simply the capacitive reactance: $Z_L = 0 - jX_c$. The magnitude and phase of the load voltage reflection coefficient are displayed as $|\Gamma_2| \angle\Phi$. Since $|\Gamma_2| = 1$, S_2 = infinity. If l and X_c have been chosen for a series resonance at the input end, then the impedance trajectory spirals in to $|\Gamma_1| \angle\pi$, and $Z_{in} = R_{base} + j0$.

Knowing the size of the top loading capacitance, V_f, and Z_0, the Smith Chart immediately tells us how long to make (i.e., H) to attain series resonance. Alternatively, given the length of the coil, the Smith Chart tells us how large a capacitance is required to bring the system into series resonance at the desired frequency. Furthermore, we can see the voltage rise from V_{min} at the driving point to a relative maximum at the load. Note that, because of the load capacitance, the line is not actually $\lambda_g/4$. Consequently, one never gets to the absolute V_{max}. One can get closer by reducing C_{top} (increasing X_c). However as the sphere gets smaller so does the disruptive potential. Clearly a trade-off should be made for the size of the machine and desired top electrode voltage.

Let us return to Equation (25) – the input impedance at the base of the extra coil when top loaded and worked against ground. For capacitive top loading, Γ_2 will be given by:

$$\Gamma_2 = |\Gamma| \angle\Phi = 1\angle\Phi \tag{26}$$

where Φ will be some negative angle.

In general, the input impedance will be

$$Z_{base} = Z_o \left[\frac{1 + |\Gamma| e^{-2\alpha l} e^{j(\Phi - 2\beta l)}}{1 - |\Gamma| e^{-2\alpha l} e^{j(\Phi - 2\beta l)}} \right] \tag{27}$$

When the structure is tuned to quarter wave (series) resonance

$$\Phi - 2\beta l = -\pi \tag{28}$$

and Z_{in} will be purely resistive (a Z_{min}). This is easily seen on a Smith Chart. Consequently, at resonance, the base impedance is given by the expression

$$Z_{base} = Z_o \left[\frac{1 - e^{-2\alpha l}}{1 + e^{-2\alpha l}} \right] \tag{29}$$

A power series expansion for the exponential gives the following rule of thumb

$$R_{base} = \alpha l \, Z_0 \tag{30}$$

which is valid for $\alpha l \ll 1$.

The above expressions for Z_{base} give the equivalent load impedance of the extra coil worked against ground only at the resonant frequency.

lt should now be clear to the reader that Tesla's 'Extra Coil' is a top-loaded transmission line resonator. As $C_T \rightarrow 0$, the electrical length should go to $\lambda_g/4$. In the lossless case, the base impedance to ground goes to zero (a short) and the top impedance becomes an open circuit load. Jordan and Balmain have presented an especially clear physical description of the resonance phenomenon on shorted quarter wave lines:

> The mechanism of resonance is particularly easy to visualize in this case. If it is assumed that a small voltage is induced into the line near the shorted end, there will be a voltage wave sent down (up!) the line and reflected without change of phase at the open (unloaded) end. This reflected wave travels back and is reflected again at the shorted (low impedance) end with reversal of phase. Because it required one half cycle to travel up and back the line, this twice reflected wave now will be in phase with the original induced voltage and so adds directly to it. Evidently those additions continue to increase the voltage (and current) in the line until the I^2R loss is equal to the power being put into the line. A voltage step-up of several hundred times is possible depending upon the Q of the line.[3]

As was pointed out earlier, if the line ended abruptly, disruptive discharge would occur from the wire sized terminal. Consequently, Tesla needed to place a charge reservoir or ball sufficiently large to not discharge at the attained voltage level. The ball itself added capacitive loading to the line and moved the load impedance from the right hand side of the Smith Chart clockwise by the angle ϕ. The length of line required for quarter wave resonance was then equal to $2\beta l$. Adler, Chu and Fano observe:

> With a small C_T in place, the line itself must store a little more magnetic energy if the whole system is to have equal amounts of both types of energy (resonance). Thus the line length must be a little less than an odd multiple of a quarter wavelength.[4]

The engineering trade-offs between the maximum desired voltage, the size of the top loading and the electrical length of the line must be seriously considered.

Examples of Extra Coil Design

In order to illustrate the above theoretical considerations, let us examine several Extra Coils actually constructed. First, let us consider the extra coil described by Tesla in the November 1, 1899 entry in his Diary.

Physical Parameters:

- D = coil diameter = 8.25 ft.
- S = turn-to-turn spacing = 1 inch
- N = number of turns = 106 (modified to 105 on Nov. 7)
- H = axial length of coil = NS = 106 inches (constant)
- d_w = wire diameter = .162 inches (#6 gauge)
- L_x = 0.02 Henries (Nov. 7, 1899)
- C_{ball} = 40 pf. (30-inch diameter sphere)
- $C_{ring} = 7.3547\pi D_{ft}/\log_{10}(4h/d)$ = 52 pf.

[3] Electromagnetic Waves and Radiating Systems, by E.C. Jordan and K.G. Balmain, Prentice Hall, 2nd Edition, 1968, pp. 226-227.
[4] Electromagnetic Energy Transmission and Radiation, by R.B. Adler, L.J. Chu and R.M. Fano, John Wiley & Sons, 1960, p. 266.

$h = 106 + 4.5 + 1$ (Aug. 26, 1899; Oct. 3, 1899)
$f_0 = 94$ KHz.
$V_{base} = 250$ KV.

These lead to the calculated parameters:

$V_f = .00428$
$Z_0 = 6380$
$Z_L = -j18.8\text{K}\ \Omega$
$\Gamma_2 = 1 \lfloor -38^\circ$
$\theta = 71^\circ$
$\Gamma_1 = 0.951 \lfloor 0^\circ$
$S_1 = 40$

On a Smith Chart, Figure 4, we enter at the normalized load reactance:

$$z_2 = \frac{-j\,X_c}{Z_o} = -j2.95$$

and advance $2\theta = 2\beta H = 142.1^\circ$ toward the generator, spiraling inward by $e^{-2\alpha l}$ to the point:

$Z_1 = .025 \qquad Z_0 = 159.5 + j0$ Ohms

For the given component of V_{base}, at f_0, Equation (20) predicts a top voltage of 9.5 megavolts. (If the wire size were reduced to .102 inches (#10 gauge) and the base voltage increased to 450 kilovolts, then $V_{top} = 11$ megavolts.) By the way, the disruptive potential for the 38 cm. radius sphere is, according to Tesla's formula, only 2.85 megavolts (Sept. 12, 1899). As a side comment, we note that the ratio of V_{base} to Zin leads to a helix current of over 1100 amps RMS – consistent with Tesla's public disclosures.[5,6,7]

On January 2, 1900, Tesla made several modifications:

N = 98 turns of #6 gauge wire
H = 98 inches
$f_0 = 88.3$ KHz
$V_{base} = 450$KV.

These lead to

$V_f = .00434$
$R_{base} = 104\ \Omega$
$Z_0 = 5197\ \Omega$

[5] "Presentation of the Edison Medal to Nikola Tesla", minutes of the May 18, 1917 meeting. Reprinted in Tesla Said, edited by John Ratzlaff, Tesla Book Company, 1984, p. 188. Also reprinted, in part, in Tribute to Nikola Tesla. Published by Nikola Tesla Museum, Beograd, Yugoslavia, 1961, p. A-107.

[6] "My Inventions - Part IV - The Discovery of the Tesla Coil and Transformer", by Nikola Tesla, Electrical Experimenter, May 1919, pp. 16, 17, 64, 89. Reprinted in My Inventions, by Nikola Tesla, Published by Skolsha Knjiga, Zagreb, Yugoslavia, 1981, p. 56; and My Inventions, by Nikola Tesla, edited by B. Johnston, published by Hart Brothers, Williston, Vermont, p. 74.

[7] "The Transmission of Electrical Energy Without Wires as Means for Furthering Peace", by Nikola Tesla, Electrical World and Engineer, January 7, 1905, pp. 21-24. Reprinted in Tesla Said, edited by J.T. Ratzlaff, Tesla Book Company, 1984, pp. 78-86. See p. 84.

C_T = 194 picofarads
V_{top}/V_{base} = 43.2

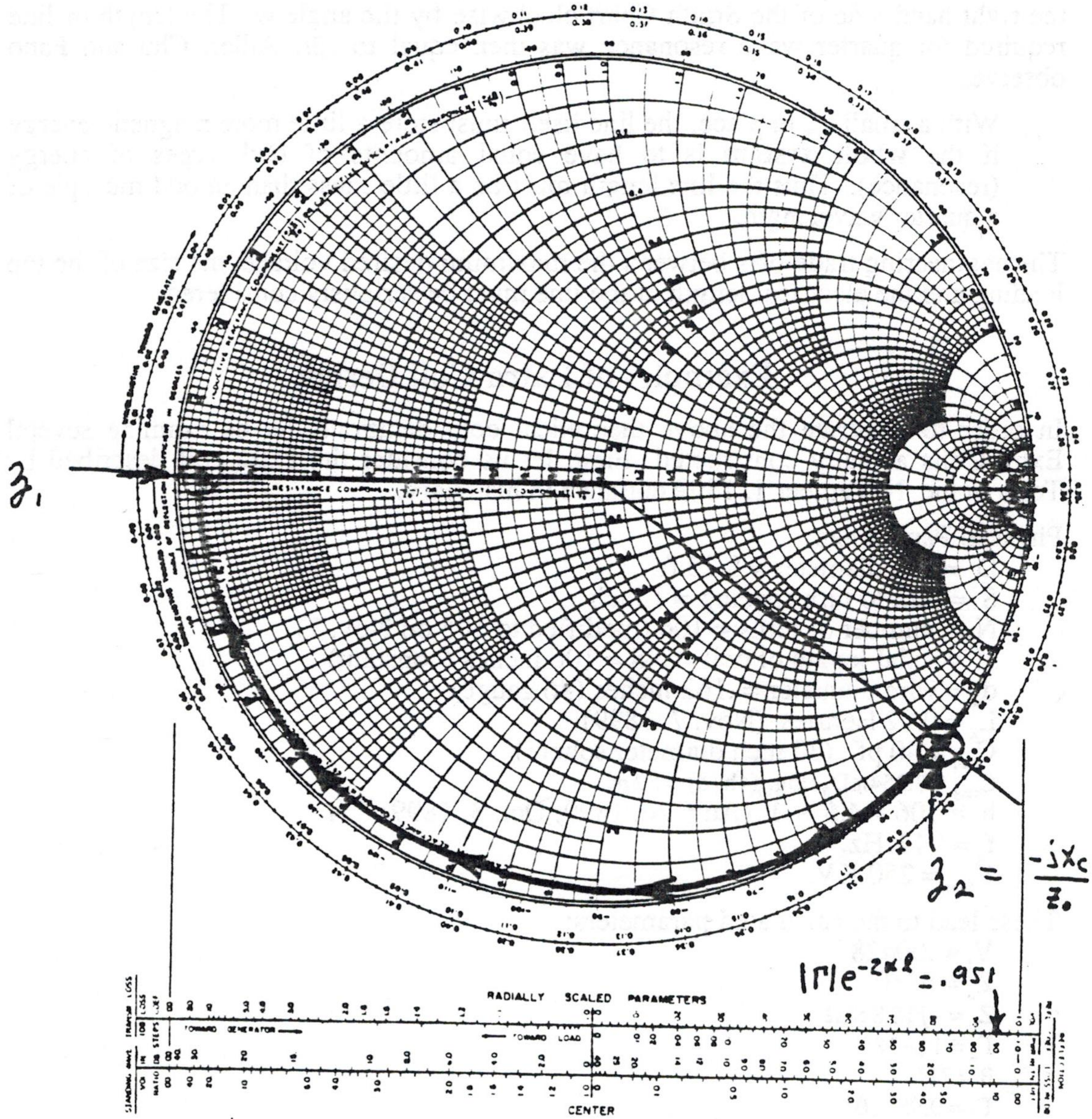

Figure 4. Smith Chart representation ofTesla's November 1, 1899 Extra CoiI.

For example, if V_{base} = 400 KV., then V_{top} = 17.3 megavolts. Other examples could be cited, and the reader may want to apply the above theory to a coil of his own design.

The standing wave voltage step-up presented above inherently presupposes simple time harmonic, or sinusoidal steady state excitation. Consequently, the theory works quite well for vacuum tube oscillator driven Tesla coils. This is called ‘forced excitation’. It should also be apparent why experimenters who found the resonant frequency of their coils by working them against ground (and not by measuring the inductance from top to bottom) were so successful.

Operation of the Extra Coil Attached to a Lumped Circuit

So far we have analyzed the behavior of the extra coil operated as a transmission line resonator when top loaded by a capacitor and driven in the sinusoidal steady state. We would now like to examine the combination of the extra coil and secondary when connected together.

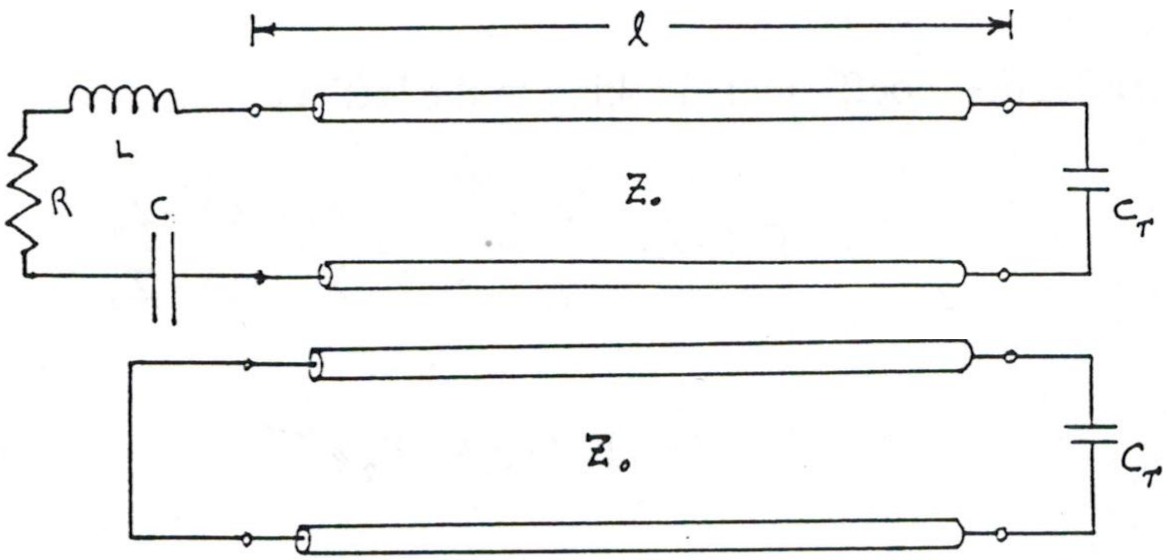

Figure 5. (a) Series resonant lumped circuit connected at the input end of a loaded transmission line. (b) Low loss equivalent

For simplicity, consider the structure shown in Figure 5, where we have modeled the secondary as a series RLC network. (It will become apparent why we used a series circuit shortly.) We desire the natural frequencies of osdillation for this system. The voltage distribution along the transmission line is given by

$$V(z) = V_{+}e^{\gamma z} + V_{-}e^{+\gamma z} \quad (31)$$

To this we must apply the boundary conditions at the ends of the line. It can be shown that the criteria for natural oscillation follow from Equation (31) and the boundary conditions as

$$Z_{+}(-z_0) = -Z_{-}(-z_0) \quad (32)$$

where

$Z_{+}(-z_0)$ = the impedance seen looking to the right at any position coordinate $-z_0$

$Z_{-}(-z_0)$ = the impedance seen looking to the left at the same position $-z_0$.

In Figure 5, we desire to have $l = \lambda_g/4$, or more precisely, to have

$$Z_{+}(-l) = R_{min} + j0 \qquad \text{(a quarter wave resonance)} \quad (33)$$

and therefore a voltage minimum (and a current maximum) at the base of the Extra Coil. Thus, we should tune the secondary coil inductance until it, with its distributed self capacitance, manifests a series equivalent resonance at the same frequency where the Extra Coil (against ground) is in quarter wave resonance (a voltage node at the base).
Adler, Chu and Fano have stated Equation (32) in an alternative manner[8]

$$\Gamma_-(-z_o) = \frac{1}{\Gamma_+(-z_o)} \quad \text{(criterion for natural oscillations)} \qquad (34)$$

where the reflection coefficient 'looking to the right' is

$$\Gamma_+(-z_o) \stackrel{\Delta}{=} \frac{V_- e^{-\gamma z_o}}{V_+ e^{+\gamma z_o}} = \frac{Z_+(-z_o) - Z_o}{Z_+(-z_o) + Z_o} \qquad (35)$$

and the reflection coefficient 'looking to the left' is

$$\Gamma_-(-z_o) \stackrel{\Delta}{=} \frac{V_+ e^{+\gamma(z_o - l)}}{V_- e^{-\gamma(z_o - l)}} = \frac{Z_-(-z_o) - Z_o}{Z_-(-z_o) + Z_o} \qquad (36)$$

But, from Figure 2,

$$\Gamma_-(-z_0) = \Gamma_1 e^{-2\gamma(-l + z0)} \qquad (37)$$

$$\Gamma_+(-z_0) = \Gamma_2 e^{-2\gamma z0} \qquad (38)$$

Thus, the input end and load end reflection coefficients are related as

$$\Gamma_1 e^{-2\gamma l} = \frac{1}{\Gamma_2} \qquad (39)$$

so that

$$\frac{(Z_L - Z_o)}{(Z_L + Z_o)} e^{-2\gamma l} = \frac{1}{\Gamma_2} \qquad (40)$$

Thus, the natural oscillations will occur (for a loss-free series resonance $\Gamma_2 = -1$) at

$$Z_L(s) = -Z_0 \tanh \gamma l \qquad (41)$$

This is a transcendental equation in the complex frequency **s**. Since the hyperbolic tangent is periodic, there are an infinite number of complex resonant frequencies s_i. As Adler, Chu and Fano observe, "With each complex natural frequency, s, there is associated a definite spatial distribution of current and voltage on the line."[9]

The lossless case of Figure 5(b) can be solved exactly. Equation (41) reduces to[10]

[8] Electromagnetic Energy Transmission and Radiation, by R.B. Adler, L.J. Chu and R.M. Fano, John Wiley & Sons, 1960, p. 210.
[9] Electromagnetic Energy Transmission and Radiation, by R.B. Adler, L.J. Chu and R.M. Fano, John Wiley & Sons, 1960, p. 210.

$$\frac{1}{j2\pi fC_T} = -j\,Z_o \tan\beta l \qquad (42)$$

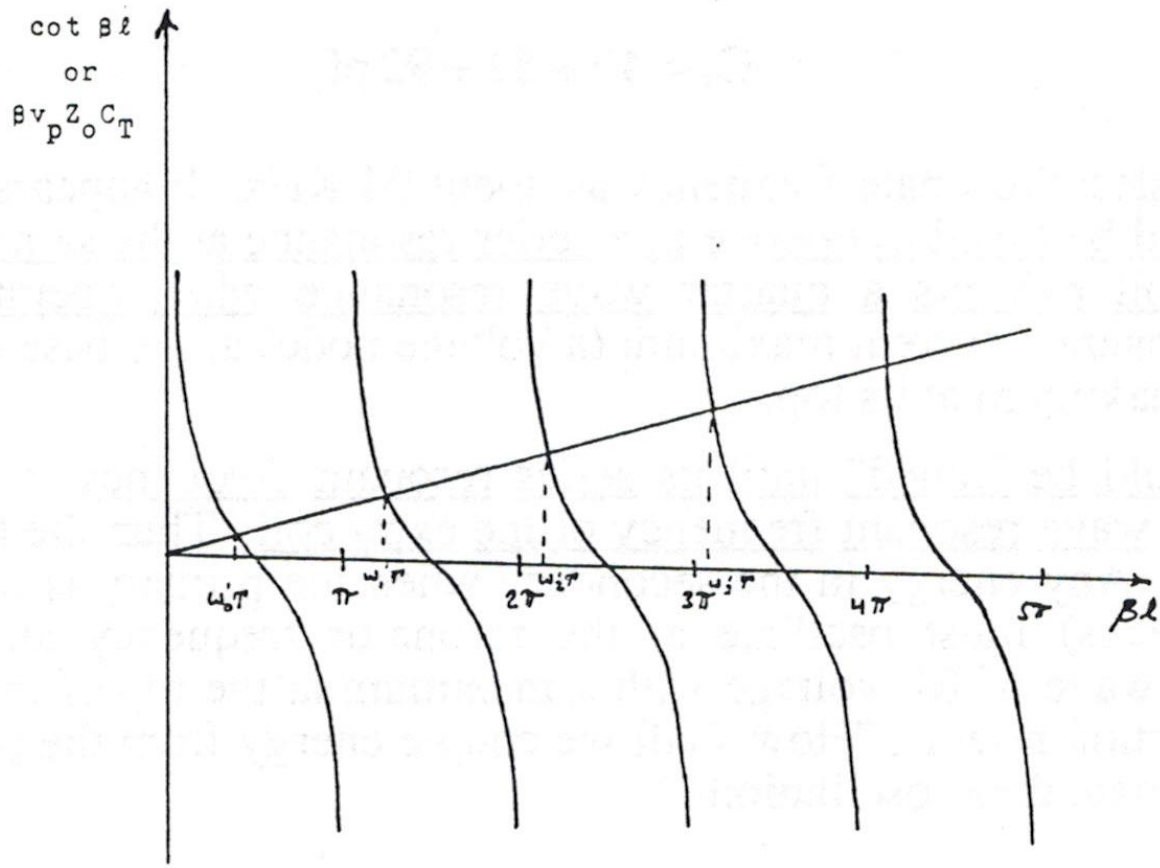

Figure 6. Graphical solution for the natural frequencies for the system of Figure 5(b).[11]

or

$$2\pi fC_T = \frac{1}{Z_o}\cot\beta l \qquad (43)$$

This equation may be solved numerically or graphically for the eigen-frequencies 2πf. Remember that[12]

$$\beta = 2\pi\frac{f}{V_p} \qquad (44)$$

Equation (42) Ieads to the resonant frequencies given by the expression

$$f = \frac{1}{2\pi C_T Z_o \tan\beta l} \qquad (45)$$

lt is of interest to consider the November 1, 1899 example of Tesla's notes, quoted above

[10] Electromagnetic Energy Transmission and Radiation, by R.B. Adler, L.J. Chu and R.M. Fano, John Wiley & Sons, 1960, p. 269.
[11] Electromagnetic Energy Transmission and Radiation, by R.B. Adler, L.J. Chu and R.M. Fano, John Wiley & Sons, 1960, p. 266.
[12] Electromagnetic Energy Transmission and Radiation, by R.B. Adler, L.J. Chu and R.M. Fano, John Wiley & Sons, 1960, p. 270.

$\beta\, l = 71°$
$z_0 = 6380\ \Omega$
$C_T = 40 + 52 + 92$ pf

These give the (lossless) resonate frequency as about 94 KHz. lt appears to us then that the secondary should be tuned to operate in a series resonance at the same frequency where the extra coil exhibits a quarter wave resonance when operated against ground. This will ensure a current maximum (a voltage node) at the base of the extra coil and a voltage maximum at its top.

The secondary should be 'tuned' until its series resonant frequency is exactly the same as the quarter wave resonant frequency of the extra coil. Then the two may be connected together. Any energy in the secondary when the primary is opened (i.e., after the spark breaks) must oscillate at the resonator frequency and therefore produce a standing wave of RF voltage with a maximum at the top of the extra coil resonator. The question now is, "How shall we couple energy from the primary into the secondary to initiate these oscillations?"

Primary to Secondary

The picture which now emerges for the operation of the Colorado Springs configuration is shown in Figure 7. Tesla operates the primary-secondary with fairly tight coupling ($k = 0.6$, which is much much greater than critical coupling). This makes sense when it is desirable to transfer energy rapidly from the primary to the secondary, i.e. for a high-power machine. It also makes sense when the primary-secondary is to work as a "transformer", and our conjecture is that the primary-secondary functioned as an RF signal generator to inject high current, at the resonator frequency, into the low impedance base of the extra coil. In another place we have discussed how triple frequencies are produced while the spark is on. However, after the spark is quenched, the secondary extra coil-system should oscillate at the natural frequency of the circuit shown in Figure 5 or Figure 6. The base voltage depends upon the voltage divider shown in Figure 8(b) and may be computed as shown. It is this voltage which is stepped up by the resonator mode.

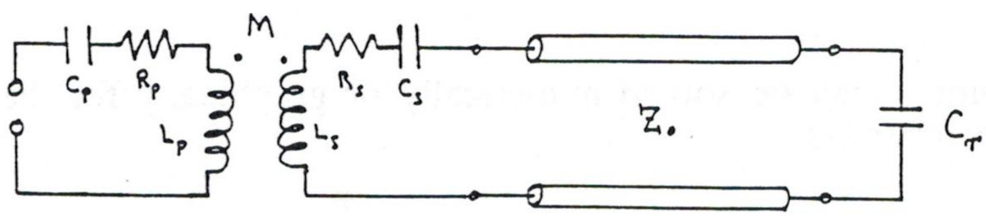

Figure 7. Equivalent circuit for the RF portion of the Colorado Springs apparatus. Note that the secondary is in series resonance at the same frequency for which the transmission line resonator is in quarter wave resonance. After the primary spark breaks, the circuit reduces to Figure 5(a).

From the above, it should be clear that the extra coil, in our view, functions like any other low-loss transmission line resonator. Consider, for example, the classical coaxial line resonator shown in Figure 9. If we replace the inner conductor by a spiral delay line, let the radius of the outer cylinder recede to infinity, and let the end to shorting plane capacitance be replaced by a sphere to ground top-loading capacitor, then the foreshortened coaxial resonator passes to a Tesla Extra Coil Resonator. The analysis is exactly the same.

Summary of Operation

Let us summarize the observations concerning our hypothetical Colorado Springs model:

1. The primary and secondary are tightly coupled ($k \gg k_c$). This can be shown to force the optimum spark dwell, t_s to very short intervals. This permits rapidly rotating breaks and therefore very high power performance for the resulting machine. This particular form of power processing is desirable for a single-turn primary machine (one with the maximum C possible).

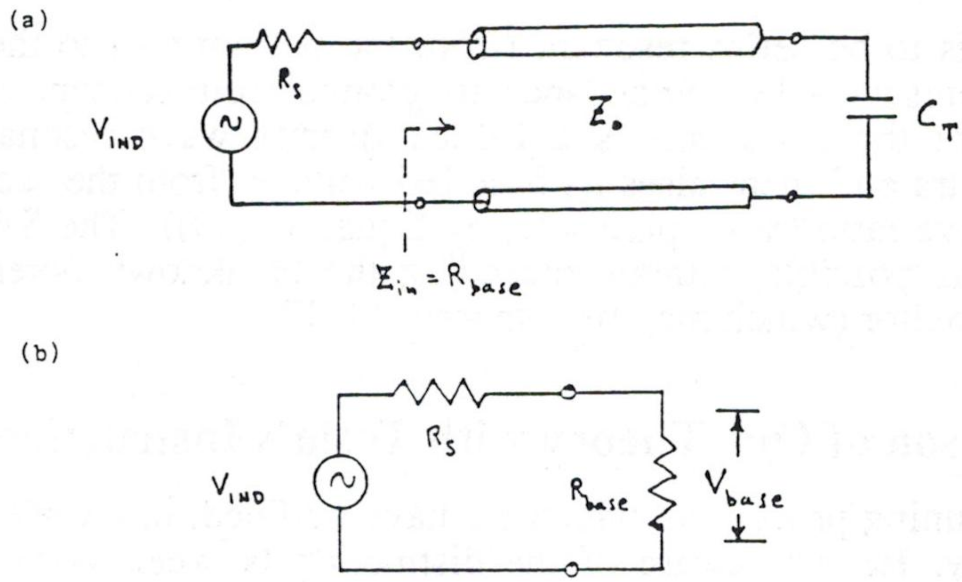

Figure 8. Equivalent Secondary-Extra Coil System during an open primary break. (b) Thevenin equivalent to obtain the transmission line input or resonator base voltage.

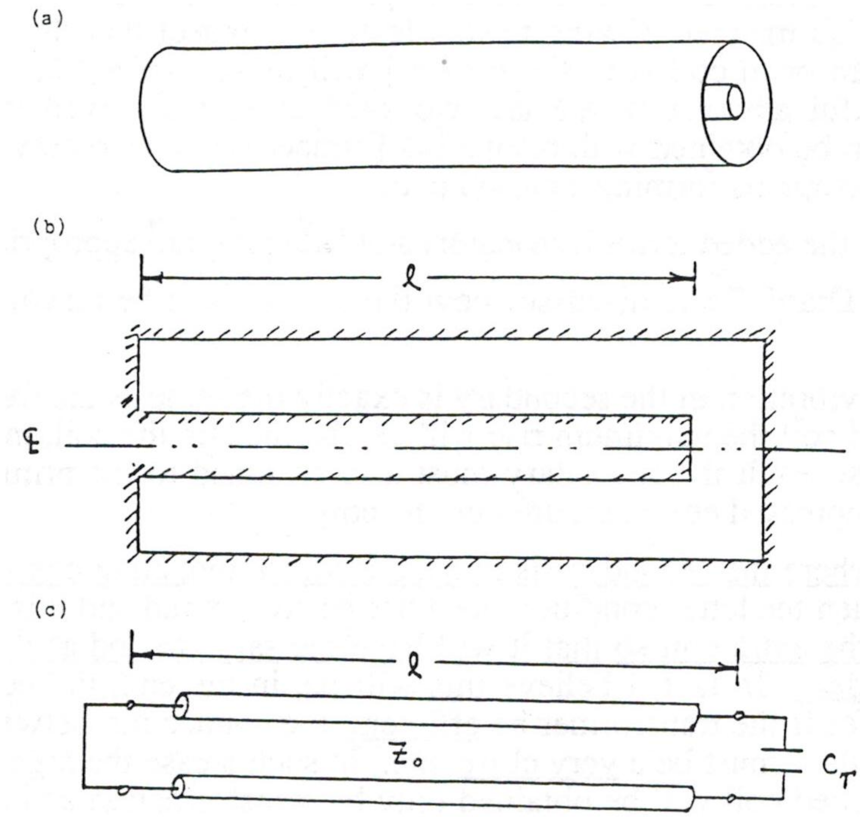

Figure 9. (a) Coaxial transmission line.
(b) A cross-sectional view of a foreshortened coaxial resonator.
(c) Its circuit equivalent.

Note: (c) is the same circuit as Figure 5(b). The analysis of the coaxial resonator is given in[13]. Equations (42) and (45) describe its behavior and consequently Figure 6 gives its resonant frequencies.

[13] Fields and Waves In Communication Electronics. by S. Ramo, J.R. Whinnery and T. Van Duzer, John Wiley & Sons, 2nd edition, 1984, pp. 504-505.

2. The secondary is to be series resonant (from the bottom turn to the top turn), and therefore present a low impedance to ground from its top, at the same frequency where the extra coil is a loaded quarter wave resonator against ground. The extra coil then raises its base fed voltage (from the secondary) by the Standing Wave Ratio (SWR) (more precisely, by Equation (19)). The SWR is to be made as high as possible without exceeding the breakdown potential of the top-loading capacitor (which may be a spherical ball).

Comparison of Our Theory with Tesla's Instructions

Tesla discusses the tuning procedure, which we have outlined, in a variety of places throughout the Diary. He was aware of the distinction between voltage rise in a lumped tuned circuit (which is proportional to the Q of the circuit) and voltage rise on an 'open coil' or resonator (which is proportional to the VSWR).

For example, in the July 10th diary entrance, he states:

> It is a notable observation that these 'extra coils' with one of the terminals free, enable the attainment of practically any emf the limits being so far remote, that I would not hesitate in undertaking to produce sparks of thousands of feet in length in this manner. Owing to this feature, I expect that this method of the emf with an open coil [i.e., a resonator] will be recognized later as a material and beautiful advance in the art. No such pressures – even in the remotest degree, can be obtained with resonating [lumped] circuits otherwise constituted with two terminals forming a closed path.[14]

We believe that the added terms [resonator] and [lumped] are appropriate.

Later on in the Diary, Tesla discusses how the system is to be tuned. On July 24th, Tesla observes:

> When the vibration in the secondary is exactly the same as the free vibrations of the excited coil the maximum rise will be obtained on the coil, in any event, but for the best result the secondary must also be tuned to the primary so that the greatest impressed emf is secured on the coil.
>
> In cases where the secondary is in such intimate inductive connection with the primary then the latter condition need not be considered and it is only necessary to adjust the extra coil so that it will have the same period as the oscillation in the secondary. In fact, I believe this will be, in the end, the best condition in practice, for if the transformer be efficient, the connection between the primary and secondary must be a very close one. In such a case the high impressed emf on the excited coil will be obtained only by transformation and not by resonant rise.[15]

lt appears to us that Tesla is saying that the primary-secondary circuit is to be tightly coupled, working to provide a lumped circuit transformer to raise the primary voltage induced across the secondary by ordinary transformer action ($E_2 = NE_1$). When the primary spark breaks, the free oscillations of the secondary are to be at the same frequency as the extra coil resonator, and the secondary is to behave as a voltage source with a secondary frequency of oscillation, f_s, equal to the natural frequency of oscillation of the extra coil, f_x.

The hypothesis concerning the relation between the primary and secondary seems to be borne out by Tesla's comments. For example, on January 1st he states:

[14] Colorado Springs Notes, by Nikola Tesla, Nolit, Beograd, Yugoslavia, 1978, p. 79 (July 11, 1899).
[15] Colorado Springs Notes, by Nikola Tesla, Nolit, Beograd, Yugoslavia, 1978, p. 103 (July 24, 1899).

… the secondary of latest type, 20 turns of two wires no. 10. The primary consisted of one turn, the two primary cables being connected in multiple arc. The ratio of conversion being thus 1:20, the emf at the terminals of the secondary, with the primary excited, to full power, was about 400,000.[16]

This is consistent with the tightly coupled coils hypothesis. For example, assuming $E_2 = NE_1$ where $N \cong k\sqrt{L_s/L_p}$ and L_s = 9 mH, L_p = 57 μH, k = .6, a primary voltage of 55,000 V would give a secondary voltage of 415 KV.

Next we consider the tuning of the Extra Coil. Tesla decided that he needed the secondary to be electrically in lumped element series resonance from top to bottom at the same frequency whereas the extra coil was in distributed circuit quarter wave resonance against ground. On September 6th, he said:

… the extra coil repeatedly described was connected in series with the secondary of the oscillator, both being tuned to the same period so that there was a nodal point on the place of connection.[17]

Again he tells us that $f_s = f_x$, We already know that the extra coil is to be in quarter wave resonance. The statement concerning the nodal point at the place of connection between the top of the secondary and the bottom of the extra coil is clarified by Tesla's sketch on page 163 of the Diary (August 26th), where he explicitly identifies the nodal point on the sketch. This clearly implies that the secondary is in series resonance – providing minimum impedance from its top to ground.

By tuning the secondary to series resonance and grounding the bottom of the secondary, it will behave as a low impedance RF source to feed the low impedance end of the resonator (the base) against the earth as a ground plane. See Figure 5(a). Many other Diary references could have been quoted, but these should suffice.

Numerical Example

Let us now calculate the overall operation of our Colorado Springs Model. We employ typical Diary values.

V_0 = 50 KV (rms) = 70,710 V (peak)
L_1 = 57 μH
C_1 = .129 μF
R_1 = 8 Ω
f_{op} = 58,693 Hz
L_2 = .009 H
C_2 = 408 pf
R_2 = 45 Ω
f_{os} = 83,056 Hz $\cong \sqrt{2} f_{op}$
M = 424.242 μH
k = .592

* If a regulating coil is included in series with the primary inductance, then one should use the 'reduced k' given by

$$k_l = k\sqrt{\frac{L_p}{L_p + L_p}}$$

[16] Colorado Springs Notes, by Nikola Tesla, Nolit, Beograd, Yugoslavia, 1978, p. 345 (January 1, 1900).
[17] Colorado Springs Notes, by Nikola Tesla, Nolit, Beograd, Yugoslavia, 1978, p. 180 (September 6, 1899).

Using the classical lossy lumped coupled coil analysis[18], we calculate the voltage induced in the secondary as V_{IND} = 383,810 volts with a double humped spectrum possessing maxima at:

f_1 = 114,820 Hz
f_2 = 52,693 Hz.

These can be manipulated considerably by changes in L_1 and M. The two frequencies would tend to imply an optimum switching time on the order of $(2\Delta f)^{-1}$ = 9.3 μseconds, as derived elsewhere.[19] The envelope of V_{IND} is actually maximum at 13.5 μs. The difference is due to the fact that the resistive loss terms are not really negligible as assumed in the $(2\Delta f)^{-1}$ formula given by Skilling.

Secondary to Extra Coil

At 83 KHz, we calculate the input-end impedance of the Extra Coil discussed in the first example above as

$$Z_{IN} = 170\ \Omega$$

with a step up of 31.8. The equivalent circuit after primary spark quenching is shown in Figure 8, where

$$V_{IND} = 383{,}810$$
$$R_s = 45\ \Omega$$
$$Z_{IN} = 170\ \Omega$$

(For maximum power transfer, one would desire Z_{IN} to be the conjugate of R_s. For maximum voltage step up, one would desire $Z_{IN} >> R_s$.) The input voltage to the resonator is then

$$V_{base} = V_{IND} \times \frac{R_{IN}}{R_{IN} + R_s} = 303{,}328 \text{ volts.}$$

The voltage at the top of the extra coil is then

$$V_{top} = 9.651 \text{ megavolts.}$$

The peak current at the base of the extra coil is

$$I_{sec} = \frac{V_{IND}}{R_s + R_{IN}} = \frac{V_{base}}{R_{IN}} = 1413 \text{ amps.}$$

This corresponds to 1000 amps RMS at the crest of the exponentially decaying envelope. This is somewhat below the 1100 amperes which Tesla reported in the January 7, 1905 issue of Electrical World and Engineer, in 1917 in the Edison Medal speech, in 1919 in "My Inventions", and in his 1934 Scientific American article. However, our numbers are only typical values reported in the Diary. If the spark duration was greater or shorter than t_s, the top voltage and base current would fall off fairly rapidly from that calculated. The numbers here, for resistive losses and frequency of operation, could probably be juggled somewhat to obtain exactly the 1100 ampere quantity if desired. By the way, taking the 8700 volts/inch rule of thumb for RF sparks, the V_{top} calculated above would produce discharges on the order of 112.5 feet. This, of course, is only relative and would actually depend upon many other parameters (atmospheric pressure, humidity, and temperature, for example).

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[18] Static and Dynamic Electricity, by W.R. Smythe, McGraw-Hill, 2nd edition, 1950, pp. 310-346. Note the error in section 9.12, equation 14, where C[1,2] should read C[2,2].
[19] Transient Electric Circuits, by H.H. Skilling, McGraw-Hill, 2nd edition, 1952, p. 235.

We also note, in passing, that the effect of the regulating coil is to vary f_{op} parameters such that the optimum dwell of the spark is equal to that produced by the given break. As Tesla often noted, the tuning should be "quite sharp".

Perhaps the most significant aspect of our theory is that it provides much more insight than the standard lumped circuit approach, which totally fails to describe the distributed resonator mode of the secondary. We hope that our analysis has convinced the reader that Sloan's discouraging 1935 comment, "… resonance transformers ... cannot be treated usefully bv mathematics."[20] is no longer valid. The remarkable work of Schelkunoff, and of Kandoian and Sichak has provided a thorough basis for the understanding of this aspect of Tesla's research. To see how these pieces fit together is perhaps the most significant advance in understanding the electrical behavior of coils of wire since Joseph Henry first discovered self-inductance in 1832. Today we can recognize that Tesla's discovery of "... the method of raising the emf with an open coil", or resonator, was certainly a "… material and beautiful advance" in the electromagnetic arts.

Sources For Figure 10

(a) "Experiments With Alternate Currents of Very High Frequency and Their Applications To Methods of Artificial Illumination" – A lecture before the American Institute of Electrical Engineers at Columbia College. N.Y. May 20, 1891. (LPA.Pe L-32). According to Tesla, a 10-KW, 10-KHz alternator was employed.

(b) A Technique employing a High Frequency alternator in the first RF experiments at the Grand Street Laboratory 1891-1892. (CSN, p, 14).

(c) "Experiments with Alternate Currents of High Frequency and High Potential", – Lecture delivered before the IEE. London, February 1892; and "Light and Other High Frequency Phenomena", Lecture delivered before the Franklin Institute. Philadelphia, February, 1893. and before the National Electric Light Association, St. Louis, March, 1893. (LPA, pp L-55 and L-112).

(d) "System of Transmission of Electrical Energy", U.S. Patent No. 645,576 applied for on Sept. 2, 1897. "Some Experiments in Tesla's Laboratory with Currents of High Potential and High Frequency." Electrical Review (NY). March 29, 1899. (Tesla Said, p. 52)

(e) "High Frequency Oscillator for ElectroTherapeutic and Other Purposes". Read before the 8th meeting of the American ElectroTherapeutic Association, Buffalo, N.Y. Sept. 13-15. 1898. (LPA, p. L-159)

(f) Colorado Springs Notes. August 17, 1899 and September 19,1899.

Comment: Parts (b) and (c) are extremely important. Without them wireless communication would have to have waited until the development of the high-power oscillator. Tesla said that by 1891 he had RF equipment "vastly superior to that of Hertz". Hertz, Lodge, Righi, Stone, Fessenden, Braun and Marconi were all using capacitive discharge in the antenna circuit (where C had to be so much smaller!). According to Stone, the latter six would not 'discover' this radio-technique for almost a decade. Clearly, Tesla's comment was not mere braggadocio.

[20] :A Radio Frequency High-Voltage Generator", by D.H. Sloan, Physical Review, Volume 47, January 1, 1935, pp. 62-70. (Note that Sloan inadvertently calls lumped coupled tuned coils 'Tesla Coils' and distributed tuned circuits 'resonance transformers'. It is quite clear that Tesla was using the former in 1891; and the latter prior to 1898! See Figure 11.

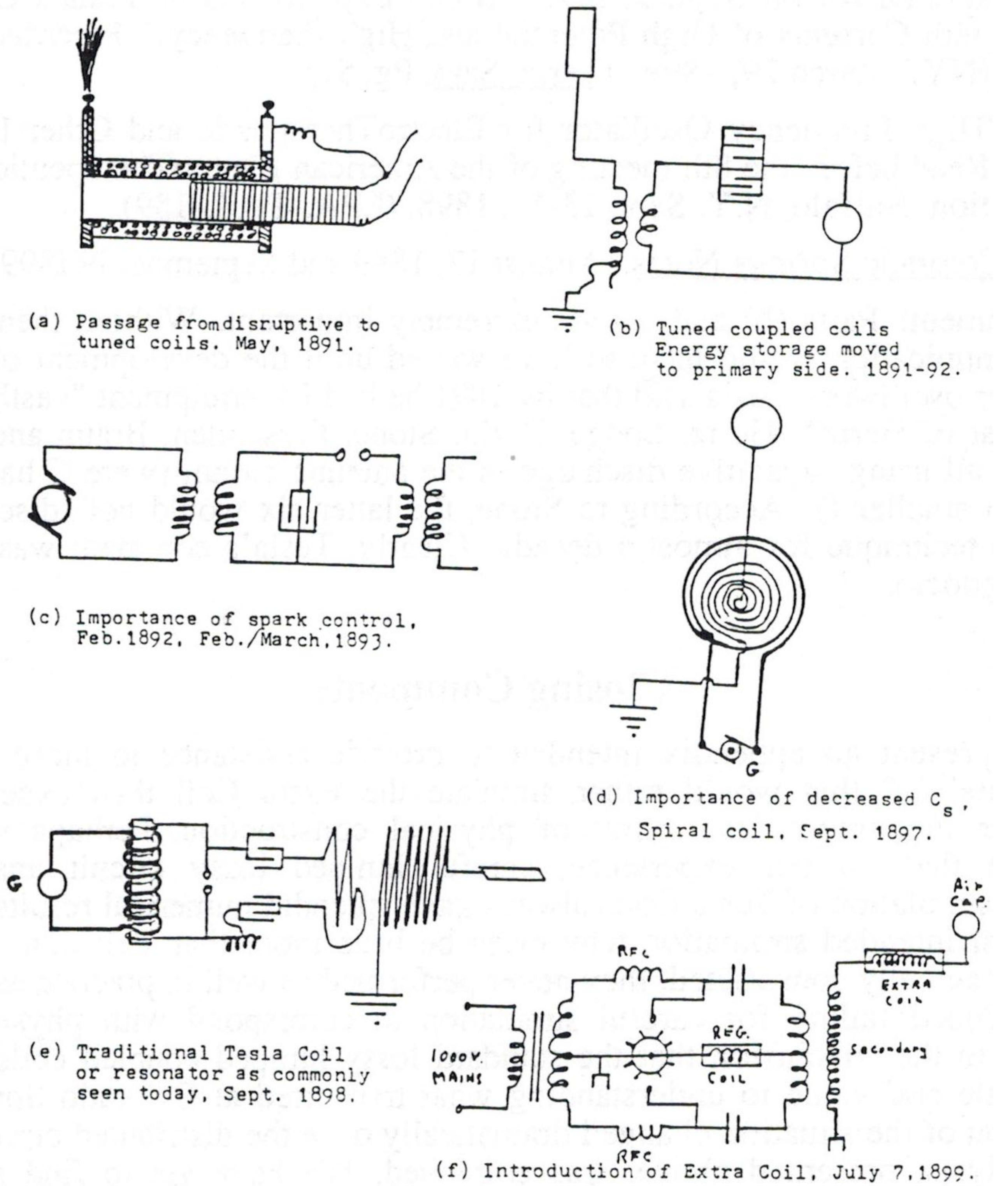

Figure 10. Evolutionary development of the Colorado Springs machine. The references are given on the previous page.

Closing Comments

We now present an appendix intended to provide assistance to those 'armchair experimentalists' that would rather simulate the Extra Coil than experience for themselves the agony (and bliss) of physical construction. Perhaps we should emphasize that, in our experience, careful lumped lossy circuit analysis and computer simulation of Tesla coils always gave splendid numerical results. However, the best intended simulation may often be little more than delusion. When the coils were actually constructed, they never performed as well in practice as on paper. This continued failure for careful simulation to correspond with physical reality forced us to the conclusion that the standard lossy lumped coupled coils approach was of little real value to understanding what transpired at Colorado Springs. The complexion of the situation changed dramatically once the distributed circuit or slow wave analysis, presented above, was employed. We have yet to find a situation where the experimental results cannot be predicted in advance with engineering accuracy.

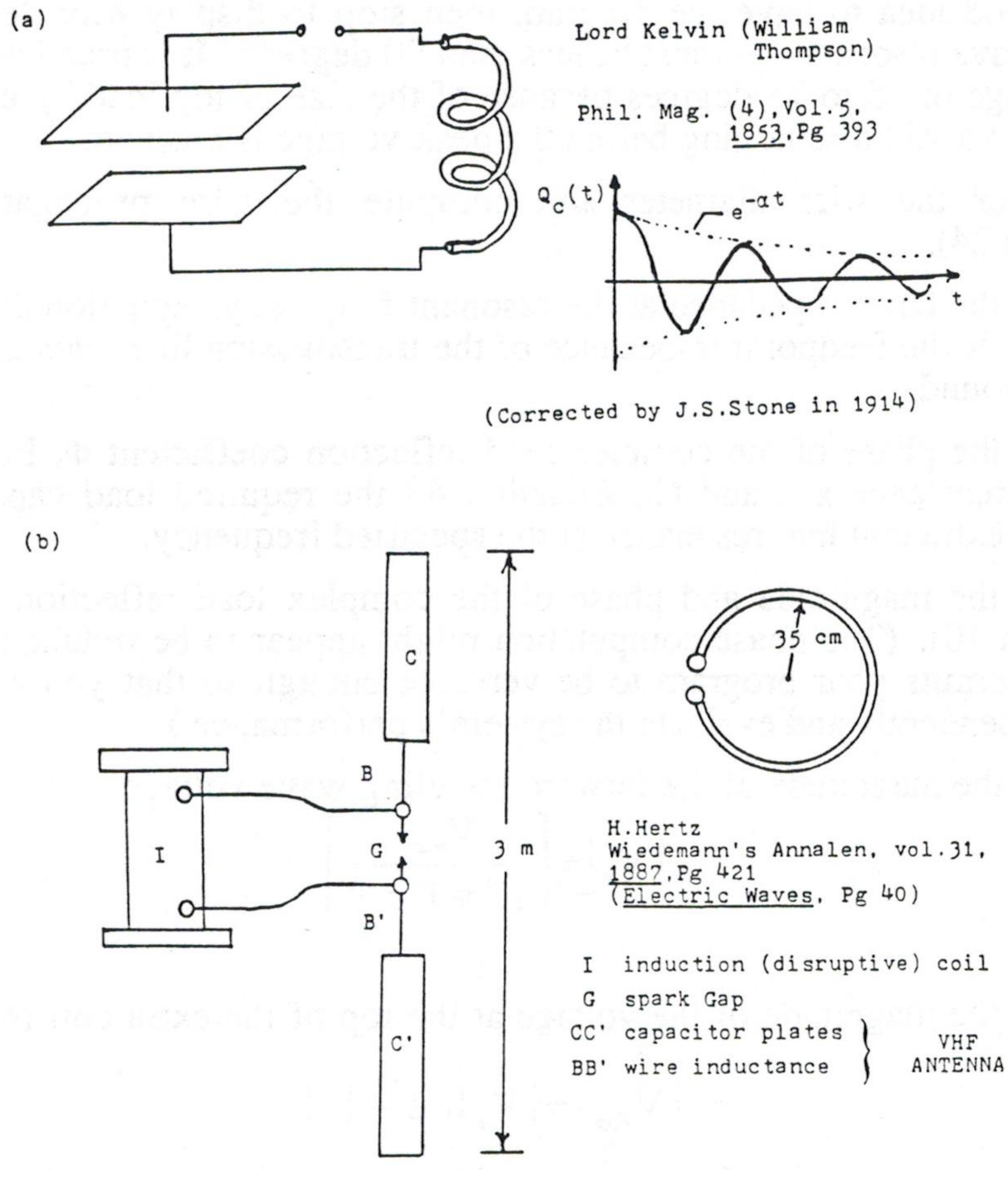

Figure 11. The development of the first RF signal generators. Note that the antenna circuit serves as both the tuning capacitance and the energy storage device.

Appendix
Extra Coil Performance Computation

This appendix outlines a simple algorithm for computing the performance of a given extra coil design. The analysis has been given above. The algorithm assumes that one knows the following input operational parameters:

D = coil diameter (in feet)
S = turn-to-turn spacing (in inches)
H = coil length (in inches)
D_w = wire diameter (in inches)
F = driving frequency (in Hertz)
V_{base} = voltage at the base of the extra coil base

The resonant rise of voltage produced by the coil and the electrical performance is calculated as follows:

1. Compute the velocity factor (Equation 1).

2. Compute the 'average characteristic impedance' (Equation 4).

3. Compute the electrical height of the coil in degrees (Equation 11).
   It is a good idea to have the program then stop to display θ in degrees. For quarter wave resonance, θ must be less than 90 degrees. It is practical to have θ in the range of 65 to 75 degrees because of the size of top loading capacitance required to avoid discharging before the peak voltage is reached.

4. Prompt for the wire diameter and compute the wire propagation losses (Equation 24).

5. Calculate the base impedance at the resonant frequency, Equation 29, and stop to display R the feedpoint impedance of the transmission line resonator worked against ground.

6. Compute the phase of the complex load reflection coefficient Φ, Equation 28, the load reactance X_c, and C_T, Equation 43, the required load capacitance to bring the extra coil into resonance at the specified frequency.

7. Compute the magnitude and phase of the complex load reflection coefficient (Equation 10). (The phase computation might appear to be redundant. However, this permits your program to be versatile enough so that you may specify C_{Top} independently and evaluate the system's performance.)

8. Compute the magnitude of the forward traveling wave voltage

$$|V_+| = \left| \frac{V_{base}}{e^{\gamma l} + \Gamma e^{-\gamma l}} \right|$$

9. Compute the magnitude of the voltage at the top of the extra coil for the given V_{base}

$$|V_{Top}| = |V_+| |1 + \Gamma|$$

   (If you cannot estimate V_{base}, then set V_{base} = 1 and step 9 will compute the voltage step up ratio.) Then stop in display V_{Top}.

Any new parameter variations may now be entered into the Program which should then return in step 1 to run the next example.

An example of an actual coil now follows.

Example

given:

D_s = 4 ft.
S_{in} = l inch
H_{in} = 120 turns ×1 inch/turn = 120 inches.
f_{Hz} = 220,000 Hertz
V_{base} = 25,000 volts
d_w = .102 inches (#10 gauge wire)

computation:

C_{top} = 19.l pf
V_f =.0102
Z_0 = 7634 ohms
θ = 78.6°
V_{top} = 953,683 volts
R_{base} = 196.1 ohms

The voltage step up is

$$\frac{V_{top}}{V_{base}} = 38.1$$

Now suppose that only one change is made. Let the wire diameter be specified as #4 gauge wire (d_w = .20 inches). In this case

R_{base} = l00 ohms
V_{top} = 1.87 megavolts
V_{top}/V_{base} = 74.8

The decreased losses produce rather dramatic results.

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The foregoing was originally published in pages 2-1 through 2-24 of the Proceedings of the 1986 International Tesla Society Symposium. The article was scanned by Gary Vesperman with minor editing such as fixing typos and replacing end-of-article references with more convenient footnotes. It was then posted on Vesperman's website www.padrak.com/vesperman as a reference by Vesperman's writings about wireless power transmission.

Famous early photographs show Tesla seated in his laboratory amidst spectacular electrical sparks emanating from the 12 million volt magnifying transformer's large extra coil – actually a giant high frequency air core induction coil.

Tesla demonstrated that his experimental Colorado Springs wireless electrical power transmission system actually worked by lighting 200 light-bulbs 25 miles from his laboratory without wires.

An unlimited number of television receivers can receive a signal within range of a single television transmitting tower. Tesla's subsequent much larger magnifying transformer in the Wardenclyffe tower at the east end of New York's Long Island was to have an output of 10,000 horsepower at 100,000,000 volts. Tesla's ultimate goal was to electrically energize global civilization with a single wireless electrical power transmission system.